TECHNICAL NOTES 2007

NATURE'S GEOMETRY: MODELING AN INFINITE FRACTAL SYSTEM WITH DISCRETE SELF-SIMILARITY [June 2007]

I . INTRODUCTION

The Self-Similar Cosmological Paradigm is primarily concerned with self-similarity that is global (applies everywhere in nature), discrete (rather than the more familiar form of continuous self-similarity), and completely unbounded (infinite number of Scales, infinite spacetime, and every class of objects is an infinite set of those objects). Furthermore, the subsystems populating each Scale are overwhelmingly comprised of ultra-compact objects approximated by Kerr-Newman, Schwarzschild and Reissner-Nordstrom black holes.

Modeling an infinite self-similar system which is dominated by an endless discrete hierarchy of ultra-compact subsystems with central singularities is going to require either (1) radically new geometric modeling methods, or (2) some very innovative applications of existing geometric techniques.

The purpose of this technical note is to organize some rather scattered and partially developed thoughts that may be relevant to this difficult mathematical challenge. My lack of talent in the realms of mathematical abstraction limits my ability to contribute much to this endeavor, but perhaps my heuristic ideas will be of some use in guiding more mathematically inclined persons toward the goal of a unified geometrical modeling of nature.

In this technical note, we will assume that the SSCP is the correct unified paradigm to supersede the current standard paradigms of cosmology, quantum mechanics and subatomic physics. The Discrete Scale Relativity form of the SSCP (see paper #12 of "Selected Papers"), wherein the discrete cosmological Scales are fully equivalent except for relative scale (i.e., exact self-similarity), is also adopted here. If nature's discrete self-similarity is less than exact, then the geometrical modeling will be considerably more complicated.

II. NATURE AS AN INFINITE UNIFIED SYSTEM

One unique feature of the SSCP is that it is a fully unified paradigm for nature. The laws of physics and the physical properties of the fundamental objects that define each Scale are identical, except for relative scale. The entire cosmos, which is infinite in both the number of Scales and numbers of subsystems populating each Scale, is one single unified system.

With the exception of a small minority of scientists such as Einstein, the overwhelming majority of physicists since 1925 have been emphasizing the concept that the atomic and subatomic realms of nature are radically different in their basic physical properties from the macroscopic and cosmic realms. If the SSCP is correct, then this "difference" is primarily due to a failure in distinguishing between how the microcosm and macrocosm *appear* and how objective reality actually *is*. Most of the leading physicists at the beginning of the 21^{st} century are openly critical of the concept of objective reality. During the development of quantum mechanics, Niels Bohr famously quipped that 'it is the business of physics to predict the outcome of measurements, not to explain how nature actually *is*.' The SSCP takes a very different point of view: that the concept of objective reality is as indispensable to physics today as it was before the advent of quantum mechanics. Furthermore, the SSCP proposes that by studying the three observable Scales of nature within the context of their inherent equivalence, we can begin again to scientifically investigate and distinguish: (1) how the different Scales *appear* to us (or other observers) and (2) how nature *actually is*, independent of *our* observations. For example, to the extent that we are able to observe and compare the physical properties of Atomic, Stellar and Galactic Scale protons, we should arrive at a very accurate unified model of the proton, given that we are able to observe it at discrete points that are equally spaced along space-time-mass scales that range over 35 orders of magnitude in length and 112 orders of magnitude in mass.

Certainly our position in nature's infinite hierarchy and the physical limits on our ability to resolve microscopic phenomena, spatially and temporally, suggest *apparent* inequalities between the Atomic and Stellar Scales. However, if we could somehow shrink ourselves and our human observing devices by a factor of Λ in lengths and Λ^D in masses, then we would discover that the physical properties of the Atomic Scale are identical to those of the Stellar Scale.

The SSCP envisions nature's infinite hierarchy in terms of a repeating pattern of subhierarchies. Each class of objects found in nature is assigned a level in the overall hierarchy. A subhierarchy extends from a base comprised of the "*elementary particles*" of Scale Ψ, to the "higher" *atomic levels* of Scale Ψ, to the very large number of *molecular levels* of Scale Ψ, up through the vast *inter-Scale levels of "condensed" objects* between Scales Ψ and Scale Ψ+1, until we reach the "elementary particles" comprising the base of Scale Ψ+1. Then the pattern just described repeats in an endless series of equivalent subhierarchies. This arrangement of an infinite hierarchy comprised of a repeating pattern of equivalent subhierarchies and dominated by discrete Scales at the bases of the subhierarchies is described in more detail in Paper #2 of "Selected Papers" section of this website.

The complexity of the amazingly unified system we refer to as nature is far beyond our capacity to comprehend in much detail (we still cannot analytically handle the 3-body problem, our supernova models fail to explode, galaxies are highly enigmatic, and exact methods are only distant dreams in fields like hydrodynamics and plasma physics). However, we can conceptualize the *general principles* of nature's overall organization, and increase the level of detail in our mental pictures by restricting our focus to limited portions of the infinite hierarchy. When one contemplates the Mandelbrot Set with its infinite levels of recursive structure, one must surely be awed. Yet the M-Set is a child's toy compared to nature. While the full complexity of nature poses an extremely daunting challenge for any form of modeling, there are at least two reasons for optimism. Firstly, like the M-Set, a relatively small number of fundamental principles and laws may underlie and largely explain the infinite complexity. Secondly, because the infinite hierarchy has the property of exact self-similarity, we should be able to combine what we learn about the Atomic, Stellar and Galactic Scales, apply this combined knowledge to studies of the Subquantum and Metagalactic Scales, and extrapolate this knowledge to regions of the hierarchy that are, and will remain, observationally inaccessible.

III. OBSERVATIONAL LIMITS AND BOUNDARY CONDITIONS

Describing the Whole System

Given a system where the number of individual levels in the hierarchy, the number of discrete hierarchical Scales, the number of subsystems comprising each level, and in many cases the number energy states available to each subsystem, are all infinite, we can expect that attempts to model the system are going to emphasize limits, approximations and boundary conditions. Any attempt at a complete mathematical representation of the entire system would quickly overwhelm our mental capacities, and even the capabilities of any conceivable computer, or set of computers working in tandem.

If we want a comprehensible understanding of the whole system of nature, then the best we can hope for is a conceptual identification and mathematical representation of:

(a) the underlying symmetry/conservation principles of the system, and

(b) the fundamental laws that determine the structures and interactions throughout the system.

As a familiar and reasonably clear-cut example of the inherent limitations on modeling and representing an infinite fractal structure, let us again consider the M-Set. One cannot generate a complete and exact graphical representation of the *whole* M-Set since it involves an endless fractal hierarchy. However, we can identify its fundamental principles, such as recursion and self-similarity, and we can state the mathematical law of the M-Set in a surprisingly succinct form:

$$z_{n+1} \leftrightarrow (z_n)^2 + c \, .$$

Given that simple equation, the definitions for its terms, and perhaps some brief instructions on how we want to represent the results of its iterations, we possess the mathematical essence of the M-Set, even if we cannot possess a complete physical representation of the whole M-Set.

The lesson is that whether we are studying the M-Set or all of nature, if we want to discuss an infinite self-similar system in a holistic manner, then we need to focus on its intrinsic principles and fundamental laws.

Describing Parts of the System

Even if we only want to model *parts* of nature's hierarchy, we still have to put stringent limits on what we include in our physical models. In conventional physics, we must define boundary conditions on the spatial, temporal and scale ranges of the physical phenomena that we intend to model. Limitations on scale, or resolution limits, would seem to be even more critical in the case of an infinite fractal since every subsystem is also an infinite hierarchical set (i.e., endless levels of substructure) and therefore any part of nature, no matter how large or small, is an unbounded fractal. As such, one must decide on hierarchy resolution limits, i.e., specifically define the limited number of hierarchical Scales and levels that are to be included in the model. An example of an extremely narrow hierarchy resolution limit would be to choose the Atomic Scale proton as the subsystem of interest and to limit the resolution solely to the proton level of $\Psi = -1$. All substructures below it would be ignored and no super-structural setting would be considered. An example of more inclusive hierarchical resolution limits might be to designate the Sun as the main subsystem of interest and include all Stellar and Atomic Scale levels of hierarchical substructure down to that of Atomic Scale electrons

Rather than assuming or hypothesizing that there are actual physical limits to nature's hierarchy (e.g., the incorrectly derived conventional "Planck scale"), the SSCP asserts that any boundary conditions on an inherently unbounded nature reflect *our* necessary but arbitrary choices of what to include, or limits on *our* ability to adequately observe nature.

Non-Differentiability

Differential equations have played an extremely important role in the history of physics. General Relativity, Electromagnetism, Thermodynamics, Hydrodynamics, Quantum Mechanics, and High-Energy Physics all rely heavily on differential equations that assume that the "manifolds" of spacetime and field geometries are *continuous* (no "holes" or "gaps" in the underlying "fabric") and *differentiable* (asymptotic smoothness in the "direction" of smaller scales). One of the founding assumptions of conventional physics is that spacetime becomes "flatter" as scale decreases and is Euclidean (actually Minkowskian, since time is included) at infinitesimal scales.

As shown by Mandelbrot, however, classical fractal structures are continuous, but ***non-differentiable***. There is no asymptotic smoothness because there is persistent "roughness" on ever-smaller scales. *The assumption of flat Euclidean geometry at small* *scales does not hold true for nature*. We can use Euclidean geometry as a rough approximation in many instances, depending on boundary conditions and hierarchical resolution limits, but exact asymptotic Euclidean flatness is only found in fictional mathematical "worlds". In the real world of nature there is very strong curvature on each of nature's endless hierarchy of discrete Scales. Therefore a new form of differential geometry of nature is called for. A key feature of this new geometry will be the ability to manage the inherent non-differentiability of nature by employing physically justified approximations and limits to the exactness of the mathematical modeling.

Consider the roughness of the Earth's surface. If one stands at the bottom of a valley in the Himalayan mountains, one is quite impressed with the roughness of the Earth's surface. However, if one observes Earth from the Moon without optical aids, the surface of the earth appears to be extremely smooth, like the cue ball on a pool table. So, is the Earth's surface rough or smooth? Answer: that depends the spatial scale and hierarchical resolution of the observational process. Although we know that the surface is definitely not ideally "smooth", one can choose to model it at large spatial scale and low resolution. We can then choose to accept the resulting smooth model as an adequate (albeit quite limited) approximation.

In principle it seems possible to retain most of the older differential geometric methods if we recognize that they cannot be exact and that they were based on over-idealized assumptions. The new differential geometric methods must recognize the necessity of imposing spatial/temporal scale and hierarchical resolution limits. This is not at all surprising, since competent natural philosophers have always felt safe in assuming that human models of nature are invariably approximations, no matter how exact and compelling the mathematical models might *seem* to be. As Einstein put it:

> "Insofar as the propositions of mathematics refer to reality, they are not certain; insofar as they as they are certain, they do not apply to reality."

My best guess is that we should retain differential geometry, but make it compatible with infinite fractals and Discrete Scale Relativity. In his book, *Philosophy of Mathematics and Natural Sciences*, Hermann Weyl made the following comment.

> "While topology has succeeded fairly well in mastering continuity, we do not yet understand the inner meaning of the restriction to differentiable manifolds. Perhaps one day physics will be able to discard it."

The introduction of fractals and the SSCP into physics brings with it the necessity of moving beyond a rigid classical "restriction to differentiable manifolds".

IV. FURTHER MODELING CONSIDERATIONS

The Evolution of Basic Geometric Assumptions

The history of natural philosophy (and physics) has been closely aligned with the search for the geometric principles that govern nature. Consider the following evolution in our basic models of nature's geometry during the modern era.

> Ptolemaic Paradigm: This was an unsophisticated Earth-centered ('what you see is what you get') model using circles, epicycles and simple Euclidean geometry to explain the apparent motions of 'heavenly objects' like planets. Stars were interpreted as 'pin pricks' in a distant dark sphere, behind which are the eternal Empyrean fires. Today this model seems very odd, and can only be seen as a major setback from the remarkably 'modern' paradigms of Democritus and the "Atomists" in the 5th century BC.
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> Galilean-Newtonian Paradigm: Building on the work of Kepler and Galileo, Newton introduced *absolute* 3-dimensional universal space and *absolute* universal time. The geometry of absolute space and time was unbounded and strictly Euclidean. This paradigm led to amazingly rapid progress, but it relied upon some questionable assumptions, such as Euclid's 5th axiom about parallel lines, the Galilean symmetry group, absolute space and time, and "instantaneous action-at-a-distance" forces.
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> Special Relativity: Einstein and Minkowski introduced the concept that space and time were inextricably intertwined in a 4-dimensional spacetime continuum. The geometry was still essentially Euclidean (with time added), but now the measurements of observers in different inertial frames were related by the Lorentz transformations, rather than the Galilean transformations. For inertial observers, absolute space and time were abolished and in their place were relative spacetime *measurements*.
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> General Relativity: In order to include accelerated frames in Relativity Theory, Einstein proposed that nature's geometry was a 4-dimensional *non-Euclidean* (pseudo-Riemannian) spacetime on macroscopic scales, although conventional General Relativity retained Minkowski spacetime as a local approximation that held for very small scales and very weak gravitational fields.
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> Discrete Scale Relativity: The recognition that nature has a discrete self-similar organization divided into discrete cosmological Scales that differ only in terms of relative scale requires a new geometry based upon discrete scale invariance. The Scale transformation equations, which relate the relative sizes of length, time and mass units on different Scales, indicate that nature is a fractal system with non-integer dimensionality associated with dynamic variables like mass and charge. So Discrete Scale relativity's main contributions to the evolution of natural

geometric modeling are unbounded global discrete scale invariance and inherent non-differentiability.

Discrete Versus Continuous Modeling

There has been a long running, and sometimes contentious, debate over whether nature is best described by continuous or discrete models. A few examples will help in defining the nature of the controversy.

An Ocean: If we are going to model the Atlantic Ocean on scales of 100 meters to 1,000 kilometers, then we would obviously model our subject in terms of a continuous fluid. However, we know that water is primarily composed of H_2O molecules and so the ocean has a fairly discrete substratum if our resolution is on the order of 10^{-8} cm. Yet if our resolution increases even further, such that we could observe individual water molecules with the same resolution with which we can observe a Stellar Scale star, then we become aware of substructures like the electronic wavefunctions and EM fields that might best be modeled in terms of continuous fluids or plasmas. That is, until we further increase the resolution so that the wavefunctions and EM fields are resolved into discrete $\Psi = -2$ plasma particles.

Atomic Scale to Stellar Scale: The bottom levels of the Atomic Scale designate two of the most discrete entities we are aware of: the electron and the proton. As ultracompact objects approximated by black holes, they are perhaps the simplest objects in nature, and are quite well characterized by just their mass, charge and spin values. This can be contrasted with the vast and unruly portion of nature's hierarchy above the molecular levels but below the first levels of the Stellar Scales. In this InterScale region of the hierarchy are cells, comets, humans, oceans, etc., which share at least one property: *they are very high N systems*, where N equals the number of "particles" composing them. When N is large, continuous fluid models seem more appropriate. But the "particles", whether they are nuclei, atoms or molecules, are highly discrete entities. Then again, even that most "elemental" particle of all, the electron, according to the SSCP has a fluid-like halo of $\Psi = -2$ particles surrounding its central singularity and its electric field can be viewed as an approximately continuous fluid of $\Psi = -2$ plasma particles. The electron's Galactic Scale analogue, a globular star cluster, provides a visual model.

A Spiral Galaxy: The Galaxy as a whole is a relatively discrete "particle", and this is made emphatic by the SSCP contention that only about 10^{-21} % of the galaxy's mass is outside of its central singularity. On the other hand, the interstellar spacetime within the galaxy would seem to require continuous spacetime modeling. And yet, the SSCP predicts that the dark matter that dominates the disk and halo of the galaxy is composed of black holes, which are sometimes regarded as discrete "holes" in the fabric of spacetime.

Transitions Between Scales: One area wherein discreteness seems most apparent is in transitions from one Scale to another. Relative units (in the Discrete Scale Relativity approach), or physical parameters and dimensional constants (in the Fixed Units approach), appear to undergo very large and discrete jumps whenever our reference system undergoes a transition in cosmological Scales. There are no protons that are intermediate in size or mass between those of Scales of Ψ and Ψ±1. We recognize clearly, however, that the concept of a transition between Scales is an abstract one and that physical objects do not change Scales, although they can be annihilated into lower Scale subsystems, or contribute mass and charge to the formation of a higher Scale object. Objects move through spacetime, and sometimes the scale of an object can grow or shrink within limits, but objects do not move through Scale.

These examples highlight the 'dog chasing its tail' quality of the continuous versus discrete quandary. Perhaps the concept of fractal systems, when we include *both* the classical fractals with continuous self-similarity (large N systems, especially common in the InterScale region of nature's hierarchy) and the newer discrete fractals with discrete self-similarity (most common at the "bottom" of a Scale), offers a third possibility. Fractals seem to point to a hybrid modeling approach that resolves the continuous/discrete conundrum by arguing: (1) that strictly discrete or strictly continuous models are artificial idealizations, and (2) that they are both useful *approximations* that apply in appropriate circumstances. Because nature's hierarchy has no "bottom", we find an endless hierarchical succession of quasi-continuous fluids, which are composed of quasi-discrete particles, which have substructures that can be approximated by quasi-continuous fluids, which are composed of quasi-discrete particles, and so on forever. Is this the final answer to the discrete/continuous issue? I would not bet on it!

Dimensionality of Space-Time-Matter

Conventional topological dimensionality, which by definition can only be characterized by integer values, and the possibility of fractal dimensionality in nature, which is usually characterized by non-integer values, have been discussed in many places, notably by John D. Barrow (*Philosophical Transactions of the Royal Society of London*, **A310**, 337-346, 1983). In that article Barrow gave several good reasons for thinking that nature's topological dimensionality involves 3 spatial dimensions and 1 time dimension (3S + 1T = 4d ST).

1. Only in 3+1 ST does the remarkable conformal symmetry of conformal geometry hold for the "source-free" (no charges or currents) Maxwell's equations of Electrodynamics.

2. There would appear to be no stable bound planetary orbits for more than 3 spatial dimensions, and this appears to be true for Newtonian physics and General Relativity.

3. In atomic physics, stable atoms appear to require less than 4 spatial dimensions.

4. The $1/r^2$ decrease in field strengths, familiar from electromagnetic and gravitational fields seems to require no more or less than 3 spatial dimensions.

So there are strong empirical reasons for thinking that the *topological* dimensionality of nature's ST is 3 + 1 = 4d. On the other hand, the SSCP shows quite clearly that the fractal or self-similar dimensionality of nature deviates from integer values. The self-similarity dimension for massive sources appears to be 3.174, since $M_\Psi/M_{\Psi-1} = \Lambda^D$ and D = 3.174. If one looks at the SSCP scaling for mass, charge, angular momentum, EM field strengths, etc., the self-similarity dimensions (i.e., the x in the scaling factor Λ^x) is usually close to 0, 1, 2, 3, or 4, but are usually non-integer in the case of dynamic parameters. Values of x for length, time, velocity and acceleration are integers (1,1,1,-1) by definition, according to the construction of the SSCP scaling rules.

The fact that D = 3.174 for mass scaling suggests that nature's topological dimensionality for source objects is 3 *spatial* dimensions, since the topological dimension is always smaller than the self-similarity or Hausdorff dimensions, but that the objects must be fractal. When time is included, the SSCP suggests that nature is a global 4d fractal with discrete self-similarity.

Modeling nature in terms of higher than 3+1 = 4d ST dimensionality is logically possible and would be justified if the assumption of > 4d ST leads to a compelling simplification in physical concepts or mathematical analysis, or if there were compelling empirical data that appeared to require it. The 4+1 = 5d Kaluza-Klein model has shown some promise as a vehicle for a unified theory of Electromagnetism and General Relativity, and the scale factor for the "curled up" 4th spatial dimension (~ 10^{-18}) is curiously close to the SSCP's ~ 2 x 10^{-18} for Λ^{-1}. Einstein was fascinated by the possibilities of the 5d K-K approach and made at least three separate attempts to use it as the basis for a unification of General Relativity and quantum phenomena. Paul S. Wesson has spent decades working on 5d models, producing numerous papers and books (*Space-Time-Matter*, World Scientific, 1999) on the subject, and feels confident that it is a very promising path toward unification, with interesting hierarchical and self-similar characteristics, and with "particles" on different scales modeled as 5d solitons.

It remains to be seen whether the 4d, 5d or some other dimensionality will turn out to be the most accurate representation of nature. Until convinced otherwise, I opt for a global discrete fractal model with 3+1 = 4d *topological* dimensionality and non-integer fractal dimensionality for actual physical systems, with a universal self-similarity dimension of about 3.174.

V. WHAT ARE THE FUNDAMENTAL DYNAMICAL LAWS

After studying nature for 35 years my guess is that gravitation and electromagnetism are the two fundamental dynamical phenomena whose interactions and attraction-repulsion balance ultimately underlie all structure and motion in nature. General Relativity (GR) and Electromagnetism (EM) are the global theories that operate on all scales and are scale invariant in their "source-free" forms. My working hypothesis is that when GR and EM are made compatible with Discrete Scale Relativity (DSR), then the coupled GR + EM field equations, which would be modified to include discrete scale invariance, would be capable of modeling all observable phenomena, at least in principle. I firmly share Einstein's intuitive conviction that GR is the correct path to unification, and that the coupled, or better yet unified, field equations of GR + EM have the potential richness and diversity to model all of nature's "unity in variety". Only a small portion of that incredible richness and diversity has been explored in depth due to the difficulties in finding exact solutions, or even approximate ones, and the complexity of the space-time-matter interactions. Improvements in numerical treatments of GR + EM, combined with better computer modeling, may eventually show the way to open up the vast unexplored territory that GR + EM + DSR seem to imply.

No doubt physics will always involve many other secondary, tertiary, … concepts, laws, rules, pseudo-forces, etc., but GR, EM and the discrete self-similarity of the DSR paradigm are the fundamental laws of nature. I suspect that all atomic phenomena, including wavefunctions, spin, photons, wave/particle duality, and entanglement will eventually be interpreted in terms of GR + EM + DSR, as will the diverse but orderly phenomena of high-energy physics.

VI. WHAT IS THE ORIGIN AND MEANING OF DISCRETE COSMOLOGICAL SELF-SIMILARITY

Scale Invariance of GR and EM

The source-free, or "vacuum", equations of Electromagnetism (EM) have full conformal symmetry. They obey all the symmetries of the 10-parameter Poincare Group and they possess a 1-parameter dilation invariance (i.e., scale invariance), as well as a 4-parameter invariance with respect to the special conformal transformations, which involve combinations of translations and inversions. Therefore the Conformal Group is a 15-parameter symmetry group.

The source-free, or "vacuum", equations of General Relativity (GR) have just two Lie Group symmetries that are *continuous* and involve infinitesimal transformations of the spacetime metric, as shown by C.G. Torre and I.M. Anderson (arXiv:gr-qc/9302033; *Phys. Rev. Lett.*, **70**, 3525-3529, 1993) . The first is continuous scale invariance, which is equivalent to dilation invariance. The second is an infinite-parameter symmetry called diffeomorphism invariance, and it is this symmetry of the 4-d spacetime manifold that gives meaning to the Principle of General Covariance, which states that the laws of physics and the intrinsic properties of any the system being modeled are completely independent of the choice of coordinate systems, such that they are independent of space, time, orientation or state of motion.

The key point so far is that *the source-free equations for both GR and EM possess continuous scale invariance*. This continuous scale invariance is a global symmetry with respect to transformations of length scales of the form:

$$L' = e^{\alpha}L\ ,$$

where e = 2.71828… is the natural logarithm base and α is a positive scaling factor. Continuous scale invariance implies that there are no intrinsic length scales.

Scale invariance is intimately related to, and sometimes synonymous with, the terms similarity symmetry, dilation invariance and self-similarity. Although these terms can be used somewhat interchangeably, there are some technical distinctions relating to their definitions, and so here we will define and primarily use the terminology: self-similarity, self-similar invariance and self-similar symmetry. Two excellent references for discussions of self-similarity and scale invariance in the context of GR are: D.M. Eardley (*Comm. Math. Phys.*, **37**, 287-309, 1974) and B.J. Carr and A.A. Coley (*Gen. Rel. Grav.*, **37**, 2165-2188, 2005).

Self-similarity can be *continuous* (no intrinsic scales) or *discrete* (a discrete set of intrinsic scales). *Geometric self-similarity* is equivalent to scale invariance and is restricted to the properties of the spacetime metric, such that lengths (L) transform as $L' = e^{\alpha}L$, metric fields (g) transform as $g' = e^{2\alpha} g$, and geometric objects or fields (Φ) transform as $\Phi' = e^{d\alpha}\Phi$, where d is the dimensionality of the object or field. *Physical self-similarity* is not restricted to the spacetime metric; it includes the geometric properties of the spacetime metric *and* the physical properties of matter fields. Written in this natural logarithm scaling format, our SSCP Scale transformation equations would be:

$$R_{\Psi} = e^{\alpha} R_{\Psi\text{-}1} \approx e^{40.8} R_{\Psi\text{-}1}$$

$$T_{\Psi} = e^{\alpha} T_{\Psi\text{-}1} \approx e^{40.8} T_{\Psi\text{-}1}$$

$$M_{\Psi} = e^{D\alpha} M_{\Psi\text{-}1} \approx e^{3.174(40.8)} M_{\Psi\text{-}1}\ ,$$

where our familiar scaling constant $\Lambda = e^{40.7926}$, and $\Lambda^D = e^{129.4757}$. Since we are trying to model nature and the actual physical subsystems of nature, we are primarily interested in *physical self-similarity*.

Where Does Discreteness Come From?

One could just say that we *observe* discrete cosmological self-similarity in nature and put that physical principle into GR and EM *by hand*, so to speak. However, it would be far better if GR and EM contained, within themselves, the theoretical and physical justification for our heuristic discovery that nature is a discrete self-similar system. I would like to suggest the following somewhat vague ideas about how discrete self-similarity *might* emerge from GR + EM solutions to the coupled GR + EM field equations.

> Working Hypothesis: Although the source-free field equations of GR and EM have continuous geometric self-similarity, when the coupled GR + EM field equations are solved *with sources of matter and charge included*, then the *stable* solutions manifest discrete physical self-similarity that is global and unbounded.

A stable solution of the full GR + EM field equations would not be a continuous set of equivalent solutions, but rather it would represent a discrete set of equivalent solutions, one for each cosmological Scale. This would be due, ultimately, to the intrinsic relativity of scale, as discussed in Paper #12 (Discrete Scale Relativity) of the "Selected Papers" section. A stable solution of Scale Ψ is also an equally valid stable solution on any other Scale Ψ ± x.

So the main idea here is that it is the restricted eigenvalue balance between the GR and EM interactions that defines stable solutions, which correspond to stable physical systems. Unstable solutions correspond to physical systems that will spontaneously decompose into lower energy stable systems.

Interestingly, in his final decade Einstein pondered the following paradox: general relativistic field theories seemed to have inherent scale invariance and yet atomic physics demonstrated that Atomic Scale systems had discrete masses and sizes. In his last scientific writing, for a 1955 relativity conference in Italy, Einstein noted that if the field equations gave $g_{uv}(x)$ as a solution, then $g_{uv}(kx)$ was also a solution, for *any* value of k. It troubled Einstein that relativistic field equations generally predicted these "similar, but not congruent" solutions, since nature's microscopic realm seemed to defy continuous scale invariance flagrantly. I like to think that if Einstein had been given enough time, and a full knowledge of the remarkable empirical discoveries that were made in physics/astronomy (like the dominating reality of Dark Matter) between 1955 and 1975, then he would have seen a delightful resolution to the paradox: *discrete* self-similarity. He probably would have recognized that, whereas continuous scale invariance is not a fundamental property of nature's physical subsystems, *discrete* self-similarity certainly is. Thus, if $g_{uv}(x)$ is a solution, then so is $g_{uv}(k^n x)$, where k is now a positive *constant* (rather than a continuous variable) and n = 1, 2, 3, … (equivalent solutions for each Scale).

What Is A Stable System?

We hypothesize that GR and EM interactions dictate that only systems with a highly restricted set of dimensional and dimensionless parameters, such as mass, radius, charge, spin, angular momentum, etc., are stable. When a stable system accretes additional energy in any form, the balance condition is violated and the additional energy is radiated away so that the system may return to a stable state. The highly collimated and energetic jets seen in pulsars, quasars, proto-stars, radio galaxies, Haro-Herbig systems, etc. are examples of de-excitation processes, which act to return these systems to groundstate stability. The global rotations and/or oscillations of variable stars, neutron stars, pulsars, active galaxies, etc. are also examples of de-excitation phenomena facilitating a return to the groundstates of the Stellar and Galactic Scale systems. Such phenomena are already well known for Atomic Scale systems. Atomic Scale systems *appear* to be more discrete than their Stellar Scale counterparts because their spatial and temporal scales are reduced by 17 orders of magnitude, and because we can study Atomic Scale systems under highly controlled conditions like extremely low temperatures and quasi-vacuum conditions.

As noted above, the stable "elementary" particles on each Scale can be combined in a limited set of ways to form a subhierarchy of atoms, a subhierarchy of molecules and an extended and sometimes chaotic subhierarchy of condensed matter objects comprising the inter-Scale segment, which extends up to the "elementary" levels of the next higher Scale.

VII. SOME FURTHER THOUGHTS ON FUNDAMENTAL PHYSICAL AND GEOMETRIC PRINCIPLES AND SYMMETRIES

No "Preferred" Reference Frames or Scales

Only nature and its subsystems are real. Reference frames and coordinate systems are artificial constructs, and therefore they cannot play a fundamental role in the true laws and principles of nature. Coordinate systems are remarkably useful for modeling purposes, but absolute or even preferred coordinate systems cannot be a characteristic of nature's most elemental principles and laws. Special Relativity taught us that space and time are not absolute, i.e., that there are no preferred inertial reference frames. Then General relativity taught us that non-inertial frames are also arbitrary and that physics must be independent of coordinate choices or imposed "background" reference systems.

Discrete Scale Relativity now teaches us that *scale*, which we had previously thought of as absolute, is actually *not absolute*. Physical scale is relative, just as are space and time. When we exclusively restrict our attention to one of nature's infinite number of Scales, then we can assign "absolute" length and mass scales *within that one Scale*. However, this in no way nullifies the more general and universal principle that for larger segments of nature's hierarchy mass and length scales are purely relative. Each of the infinite number of Scales is completely equivalent, and so there can be no justification for absolute or preferred scales. The property that unifies nature's hierarchy is its discrete self-similarity, which obeys the same universal Scale transformation equations throughout the hierarchy.

Some Corollaries to Relativity of Scale

(a) If our currently fashionable physical theories involve absolute space-time-mass scales, or absolute "background" spacetimes, then they are not truly fundamental. At best, they only apply as an approximation within the confines of one Scale.

(b) In modeling nature it would seem best to use relative space-time-mass units that are directly derived from the physical properties of actual physical subsystems. For example, the proton radius or the wavelength of a specific atomic transition would seem more appropriate than the centimeter as a measure of length. We tend to choose units that are related to our particular size scale, but this is not a necessary strategy and it may be considerably less than optimal.

(c) One interesting speculation based on the infinite hierarchy paradigm concerns the concept of inertia, which remains something of an enigma to this day due to the uncertainties over whether the initial hopes that General Relativity might completely resolve this issue have been fulfilled. In an infinite hierarchy of discrete subsystems it would seem that any object is simultaneously traveling in an infinite number of different directions, with an infinite number of different velocities between 0 and c. To see this more clearly, consider a person sitting in a chair on Earth. He has separate motions due to the spin of the Earth (~1,000 mph), the orbit of the Earth (~35,000 mph), the motion of the Solar System in the Galaxy ($\sim 10^2$ km/sec), the motion of the Galaxy in its local galactic environment ($\sim 10^6$ mph), and so on in an endless series of different motions on ever-larger scales. If we see each motion as a vector with its tail end at our observer's chair, its length proportional to the velocity of the motion, and the direction fixed with respect to some arbitrarily chosen physical frame within the hierarchy that is arbitrarily high "above" the observer's scale, then we get a vast number of different vectors pointing in a random, spherically symmetric distribution, with an arbitrarily uniform distribution of lengths. Might this idea help to explain how one can get a state of apparent "rest" in a purely relative system? Or has Einstein's General Relativity already answered that question? I leave it to the physicists to decide if the infinite array of velocity vectors idea might be useful in improving our understanding of the concepts of inertia and rotation.

(d) Recursion and self-similarity are intimately related. Where you find one, you are almost certain to find the other. Most classical fractals, such as the M-set or Cantor's "dust", are generated by recursive iteration of an equation or physical process. Nature's infinite hierarchical system is eternal, and so it is not *generated* by a recursive process. However, the following scenario shows how discrete recursive accretion might "anneal" temporary and highly localized "gaps" in nature's hierarchy.

We start with Stellar Scale electron and positron analogues heading towards each other and undergoing an annihilation reaction. We hypothesize that all Stellar Scale structure is converted into $\Psi = -1$, or Atomic Scale, subsystems. Subsequently, if $\Psi = -1$ electrons and positrons collide, then they too could annihilate into $\Psi = -2$ particles. This cascade of annihilations would seem to be reversible under the right physical conditions and that recursive gravitational accretion might spontaneously take place. If enough $\Psi = -2$ particles are compressed into a small enough volume, then they could gravitationally coalesce into $\Psi = -1$ particles, which could subsequently be forced to coalesce into Stellar Scale particles.

This recursive "annealing" of local "gaps" in nature's hierarchy is reminiscent of the current conventional ideas about how the observable portion of our Metagalactic environment evolved from a state of "pure energy" ($\Psi = -2$ particles?) to the formation of Atomic Scale particles, and to the subsequent formation of Stellar and Galactic Scale systems. Reforming the local properties of nature's hierarchy may not always need to follow a strict recursive sequence, but might also proceed simultaneously on several Scales. We are a very long way from working out the details of such a complex phenomena as multi-Scaled annihilation and annealing of nature's hierarchy, but it is worth mentioning as a future topic of study.

Principle of Maximum Symmetry

The evolution of physics is closely related to the evolution of our understanding of the symmetries of nature. Major advances along these lines were achieved by Sophus Lie who developed the foundations of Group Theory, which identifies, organizes and classifies the symmetries of mathematical systems. Another notable advance arose from the work of Emmy Noether, who proved that for every continuous point symmetry there is a specific conservation law in physics. As we have increased our understanding of the physical world, we have found broader, deeper and more varied symmetries at the fundamental level of things.

Modern physics started with the Galilean Group: a 10-parameter group defining symmetries with respect to 3 spatial translations, 1 temporal translation, 3 rotations and 3 velocity transformations. Then Special Relativity necessitated a transition to the 10-parameter Poincare Group: 4 translations in space-time, 3 rotations in space, and 3 Lorentz "boosts". Adding dilation invariance defined the 11-parameter Relativistic Similarity Group, which is sometimes called the Weyl Group. Maxwell's "vacuum" equations of Electromagnetism demonstrated that nature sometimes employs the 15-

parameter Conformal Group: the 10-parameter Poincare transformations, plus 1 dilation and 4 special conformal transformations. Then came General Relativity with its infinite-parameter Diffeomorphism Invariance. Since that time many types of symmetries have been applied in various branches of physics: continuous and discrete, broken and unbroken, classic geometric symmetries and abstract heuristic symmetries.

A review of 'Symmetry In Physics Since 1905' would require several huge tomes and would divert us from our main purpose. Our primary interest here is pursuing the important question: what new ideas does Discrete Scale Relativity bring to the study of nature's fundamental symmetries? My current best candidate for the answer to this technical question is: global discrete scale invariance. The scaling equations of the SSCP tell us that this invariance applies to *everything*, which is modeled in terms of the elementary dimensions of space, time and mass/energy, as well as the derived parameters such angular momentum or charge. When global discrete scale invariance is a fundamental symmetry of nature, then the deterministic dynamic equations of the elementary interactions (GR + EM) maintain the discrete cosmological self-similarity of nature. In slightly different words, ***the new symmetry is discrete scale invariance and the physical embodiment of that symmetry is the discrete cosmological self-similarity of nature's physical systems.***

Since classical symmetries involve transformations that leave a vector, object or field unchanged, one should clarify exactly what is being transformed in Discrete Scale Relativity. Clearly, Atomic Scale atoms are not being transformed (expanded) into Stellar Scale atoms (e.g., stars). Rather, the size of the units of measurements (relative to the observer's Scale), the sphere of observation and the hierarchical resolution are being transformed. We can look at the Solar System with a scale resolution range of, say, 10^6 cm to 10^{16} cm, and we can, in principle, observe one of its excited Li atoms in an analogous Rydberg state with a scale resolution range of 2×10^{-12} cm to 2×10^{-2} cm. The scale of observation is transformed, but we observe equivalent physical systems with wildly different sizes.

If we allow the radical concept of observers on all Scales, then we can say that the observers on any Scale will observe equivalent "observable universes", with the same types of physical systems and the same dynamical laws. It is a lot easier to grasp translational or rotational symmetries. Discrete cosmological scale invariance is a bit more abstract and considerably more counterintuitive. Discrete cosmological Scale symmetry is global, unbounded, and covariantly applies to both the geometric properties (e.g., lengths) and physical properties (e.g., mass and charge) of nature's subsystems.

Since my understanding of diffeomorphism invariance is still very limited, I am uncertain of the relationship, or lack thereof, between it and discrete Scale symmetry. Perhaps nature's discrete scale invariance picks out a highly restricted subset of stable solutions from the apparently infinite number of different possibilities implied by the diffeomorphism invariance, continuous scale invariance and conformal invariance of the "vacuum" forms of GR and EM. I leave this question unanswered and hope for technical assistance from topologists, fractal geometers and theoretical physicists.

Quite possibly the terms mass and energy are conceptual shorthand for something more closely associated with the geometry of space-time-scale. I suspect that Discrete Scale Relativity might revive the ancient dream shared by Riemann, Einstein and many others of a purely geometric understanding of physics, with no *ad hoc* pseudo-fields, just-so phase transitions or other questionable theoretical contraptions tacked on.

A New Conservation Law?

Noether's theorem demonstrated an intimate relationship between continuous point symmetries and conservation laws. For example, the Galilean Group of space and time symmetries is linked to conservation of momentum and energy, respectively. It seems natural to ask whether discrete cosmological scale invariance, i.e., Discrete Scale Relativity, is coupled to some new conservation law.

Briefly, the answer to that question seems to be: probably not. The main reason is that in the case of discrete cosmological scale invariance we are dealing with a discrete symmetry principle, and that would seem to preclude a coupled conservation law of the conventional Noether's theorem type. It is conceivable that just as discrete cosmological scale invariance is an unorthodox symmetry principle, so it might be coupled to a new conservation law by an unorthodox coupling theorem. Only time will tell.

Determinism and Causality

Like Spinoza and Einstein, I am a firm believer in strict causality and in a universe that is fully deterministic. As we have learned over the past 90 years by studying chaos, fractals and dynamical systems, the deterministic behavior of nonlinear systems includes abrupt changes, strange attractors and sensitivity to initial conditions. These properties can mimic "randomness", even though the underlying physical processes are completely deterministic. If the truth be told, all interactions are probably nonlinear when the resolution is fine enough and all variables are included. We often mistake linear approximations for accurate physical laws. Solar System studies underwent a classic shift in modeling assumptions from a focus on simple classical mechanics to a more complete dynamical systems approach. As Einstein noted: 'The real equations of nature are nonlinear'.

I suspect that the indeterminism built into the mathematics of Quantum Mechanics will eventually be recognized as an artificial first approximation that works fairly well but is not correct in terms of fundamental principles. I expect that the study of fractals, chaos, dynamical systems, and Discrete Scale Relativity will lead us back to a fully deterministic physics that explains observed quantum mechanical properties of atomic systems in terms of more sophisticated approximations grounded in nonlinear dynamical systems theory. Although the laws of physics would be completely causal and deterministic, the complexity and interconnectedness of nature severely limit what can be reliably predicted, and how far into the future those predictions can extend (planetary motions: ~10^3 years; weather : ~ 4-10 days).

Such a major paradigmatic transition will not take place quickly, nor will it occur without a momentous intellectual struggle.